CTE Model for Estimating CCD Image Smear

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Abstract

Radiation promotes charge transfer inefficiency (CTI) in a CCD, causing focused images to become smeared. For example, such smearing will degrade the accuracy and precision of a CCD-based pointing system. A model has been created whereby CTI smearing caused by a radiation induced traps can be projected, given the spatial distribution and pixel position of the image, the large signal CTI, temperature, parallel and serial transfer frequencies. It is well known that CTI increases with signal size to some asymptotic level, and the model description offers a theory and function for this behavior. The final product indicates several complicating factors affecting CTI evaluation which could inflate results, and make precise comparisons between experiments difficult.

1.0 Introduction

CCDs are often considered for precision measuring systems in space. An ever present obstacle against this purpose is the relatively hostile radiation environment there. Theoretically radiation should degrade most operational facets of the CCD, but it primarily affects dark current and charge transfer efficiency (CTE). [Inefficiency, i.e. CTI = 1 - CTE, is also commonly used.] Such effects have been frequently cited in publications. This report is focused on projecting the effects of radiation on image smear via CTE degradation.

A particular problem arose which involved pointing CCDs. In this case, an algorithm extracts a position by determining the best fit of a chosen distribution to a finite set of signal points associated with an objective, e.g. a star. The best fit indicates a center for this sample distribution. As such, charge loss from the original to the trailing pixels will result in a relative shift in the center position and subsequently increase the uncertainty in the actual image location, i.e. pointing accuracy and precision will be impaired. Furthermore, this CTE smear will lengthen with the number of parallel and serial transfers required to transport the image to the output. The dependence of the CTE smear on image location produces pointing distortion. Undoubtedly the noise associated with the transfer inefficiency itself will also affect pointing precision, but it was assumed to be a secondary factor, and so has not been examined yet.

2.0 Models

2.1 Trap and Emission

It seems common to conceptualize trap behavior as a case where charge occupies some amount of pixel volume, which also contains some traps. Mohsen and Tompsett described the dynamics of this situation by the equation

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$$\frac{dN_f}{dt} = \frac{N_t - N_f(t)}{\tau_c} - \frac{N_f(t)}{\tau_e} \qquad (1)$$

where N_f is the filled trap density (treated here as counts per pixel), N_t is the trap density, and τ_c and τ_e are the capture and emission time constants respectively [1]. The general solution for $N_f(t)$ is

$$N_f(t) = \frac{N_t}{(1+\tau_c/\tau_e)} \cdot \left\{1 - \exp\left[-t \cdot \left(\frac{1}{\tau_c} + \frac{1}{\tau_e}\right)\right]\right\} + N_f(0) \cdot \exp\left[-t \cdot \left(\frac{1}{\tau_c} + \frac{1}{\tau_e}\right)\right] \qquad (2)$$

where

$$\tau_c = \frac{1}{\sigma_n \cdot v_{th} \cdot n_s} \qquad \text{and} \qquad \tau_e = \frac{g}{\sigma_n \cdot v_{th} \cdot N_c} \cdot \exp\left(\frac{E_t}{k \cdot T}\right). \qquad (3)$$

Here σ_n is the capture cross section, v_{th} is the average thermal velocity of the charge carriers, n_s is the signal charge density, g is a level degeneracy factor, N_c is the density of conduction band states, and E_t is the trap energy level. The maximum for τ_c will occur under the minimum value for n_s. Using Eqn. (8) [to be described later], n_s was found to equal 2.017e15 for 1 electron and 2.037e15 for 100 electrons, so the evaluation of τ_c and τ_e in Table 1 below should be suitable for low signal applications in general.

Tmpr (^oC)	τ_c (nsec)	τ_e (msec)	Tmpr (^oC)	τ_c (nsec)	τ_e (msec)
+ 20	10.2	0.02	- 60	12.0	19
0	10.6	0.08	- 80	12.6	253
- 20	11.0	0.37	-100	13.3	5818
- 40	11.5	2.27	-120	14.1	29530

Table 1: Capture and emission time constants versus temperature for 0.42 eV traps in a silicon CCD with a dim signal image, according to Eqn. (3).

Equation (2) may be simplified by declaring an effective trap density, labeled N_t', as being equal to $N_t / (1 + \tau_c / \tau_e)$, and defining an effective time constant τ_{eff} as

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_c} + \frac{1}{\tau_e}.$$

Now $N_f(t)$ can be expressed as

$$N_f(t) = N_t' - \left(N_t' - N_f(0)\right) \cdot \exp(-t / \tau_{eff}). \tag{4}$$

This equation can be simplified further by defining two more variables N_{av} and "cap". N_{av} is defined as the number of available trap states per pixel, i.e. $N_t' - N_f(0)$. Capture count (cap) is described as $N_f(t) - N_f(0)$ The interesting feature about both variables as just defined is that it implies the possibility of a negative number of available trap sites and capture counts. This anomaly is really just a matter of semantics. For example, N_t is defined by the pixel volume occupied by the signal charge. A decrease in signal means a reduced signal volume. By the accounting method used here, N_t would therefore also decrease, but $N_f(0)$ would be based on the larger predecessor, so N_{av} could thus be negative.

Subtracting $N_f(0)$ from both sides of Equations (4) and inserting the aforementioned substitutions produces

$$cap(t) = N_{av} \cdot \left[1 - \exp(-t / \tau_{eff})\right] \tag{5}$$

When N_{av} is positive, charge is captured and the signal suffers a net loss. When N_{av} is negative, charge is emitted, i.e. the "capture rate" is negative.

2.2 Signal Volume

Holland recently gave a good explanation of how the signal volume will approximately vary with its charge count [2]. He noted that thermal energy will cause a single electron to occupy a disproportionate amount of volume compared to what it would be for large charge counts, e.g. "the volume occupied by 1 electron is the same as that occupied by 1000 electrons." Such a rule may be too gross for modeling trap behavior at small signal levels, based on the observations that a fat zero level on the order of 100 electrons improves CTE measurably. The issue here is trap accessibility, which judging from Figure 1 [from Ref. 3], is a monotonic function of signal size and - by implication - signal volume too. So a better representation with regards to traps may be that portrayed by Mohsen and Tompsett [1] who estimated volume from solving the one-dimensional Poisson equation. The relationship can be empirically described very well by

$$Vol = V_o \cdot \left[1 - \exp(-Sig / S_o)\right] + Sig / n_d \tag{6}$$

where "Sig" is the signal charge count and n_d is just some constant, although it was chosen to suggest a strong dependency on buried channel doping density. The graphical representation of this volume dependency is shown in Figure 2 along with the implied interpretation for V_o as the thermally disproportionate volume for 1 electron. S_o is essentially the upper signal level for V_o.

To see how this model effects a CTE estimation, let us imagine a serial register which is N_{pix} long. It could be flooded by a continuous signal train where each pixel is nearly filled to its full well capacity. With this condition, all the usable volume and all of the available traps are being occupied. The sum of deferred charge (N_{def}) should then be about

$$N_{def} = n_t \cdot Vol|_{fw} \cdot N_{pix} \ . \tag{7}$$

where n_t is the volumetric trap density and $Vol|_{fw}$ is the volume of a full well signal. Near full well, a (good) CTE can be approximated by [3]

$$CTE|_{fw} = 1 - N_{def} / \left(N_{pix} \cdot Sig|_{fw}\right) \ ,$$

where the subscript "fw" designates the full well state. Substituting in the expression for N_{def} produces

$$CTE|_{fw} = 1 - \left(n_t \cdot Vol|_{fw} / Sig|_{fw}\right) \ .$$

Eqn. (6) implies that at very large signal sizes, the ratio Vol /Sig should approach $1/n_d$, so

$$CTE|_{fw} = 1 - \left(n_t / n_d\right)$$

or

$$N_t = n_d \cdot Vol \cdot CTI|_{fw} \ . \tag{8}$$

This equation estimates the trap count per pixel for any signal size, assuming a relatively uniform trap density, although the accuracy relies upon how small V_o [Eqn. (6)] is relative to the full well volume. This question will be addressed later. For now Eqn's (6) and (8) will be substituted into the expression for N_{def} in Eqn. (7) to produce

$$N_{def} / N_{pix} = CTI|_{fw} \cdot n_d \cdot V_o \cdot \left[1 - \exp(-Sig / S_o)\right] + CTI|_{fw} \cdot Sig \tag{9}$$

From the large signal perspective, Eqn (9) behaves like

$$N_{def} / N_{pix} = CTI|_{fw} \cdot n_d \cdot V_o + CTI|_{fw} \cdot Sig$$

or N_{def} = intercept + slope * Sig

like the straight line extraction seen in Figure 1 from Janesick [3]. Notice that the Figure 1 plot indicates that the slope in the large signal regime gives the full well CTE. Eqn. (9) suggests that the intercept position is also linked to the slope through the full well CTI, but is also dependent upon V_o. Subsequently, the intercept implies the number of available trap sites associated with V_o. To match the Janesick plot, V_o and S_o were estimated to be 2.1 μm^3 and 4.7K e-'s respectively. Near full well, which is about 200K e-'s for some CCDs, the volume was estimated to be about 10 μm^3, so the assumption made to Eqn (6) to simplify Eqn (8) was reasonable.

2.3 Model Algorithm

The CTE smear algorithm incorporates the expressions for trap emission and capture rates plus volume variation with charge count as discussed above. Some variables like cross section have been set constant to values reported in the references for the purpose of evaluating the model. Precise work would require careful characterization. A single trap energy level of about 0.42 eV was assumed throughout this study. This effect assumes that transfer inefficiency is dominated by bulk radiation damage. [2]

The algorithm starts a loop where each iteration calls for a pixel transfer. At the start of each iteration the volume associated with the entire signal is calculated through Eqn (6). Then the total number of traps per pixel and what is captured or emitted are calculated with the additional help of Eqn's (8) and (5) respectively. What is not trapped is considered to be mobile and therefore is subsequently advanced one pixel with respect to the trapped train in preparation for the next iteration. After the signal train reaches the output, the first mobile pixel is simply stored in an output array instead of having it advanced. The result for modeling 521 transfers at 50 kHz and -80 °C with a CTI of 6e-7 is shown in Figure 3 which matches Figure 1 pretty well. The effect of smearing on a spot is shown in Figure 4 where a poor CTE value, suitable for an irradiated CCD, was employed. Verifying these projections with actual devices and test conditions is under way. Other model results are offered in the next section.

3.0 Discussion

A product of this study is a CTE model. The model tracks trap capture and emission rates based on signal sizes, particularly its volume. The effect links small signal CTE to large signal CTE in a continuous distribution versus signal. On an operational scale, the model tracks this charge exchange with traps over a selected number of transfers. The parameters from the model expression are provocative. They suggest that, at large signals, deferred charge is proportional to the signal size because the charge density is constant, estimated by n_d. However as the signal decreases below S_o, thermal energy allows charge carriers to encounter a disproportionately larger amount of traps than it "should". This is obviously a crude model. For example, the effects of temperature on V_o has been omitted for now.

The model assumes a uniform density of traps, but that concept constitutes a major issue. It is the nature of traps to be discrete, and in high quality channels there would be too few of them to constitute a uniform density. This phenomena apparently can be seen in charge pumping maps and Fe-55 CTE plots when the trap size is large enough to be obvious, i.e. more than about 20 e-'s. Conversely, Fe-55 plots usually shows a smooth degradation slope. This might indicate a uniform distribution of small traps, but it might also indicate that charge is typically not deferred by interacting with traps.

It might be better to think in terms of the number of traps in a core volume which runs the length of a register where the volume expands or contracts with charge count. Figure 1 shows a N_{def} intercept of 13, and a N_{def} of 33 for a signal level of 65000 e-'s/pixel distributed along 521 pixels with a CTI of 6e-7. Therefore, for a CTI of 6e-6, N_{def} should be on the order of hundreds. Subsequently the volume is directly linked to the probability of any electron running into a trap. In any case, this model assumes that one type of trap characterized by a single energy level dominates the CCD's CTE, and that the traps are uniformly distributed along a register. As such, it is probably best suited for irradiated devices, which is how it is being used now.

It is apparent that comparing published CTE results is difficult because of the many factors which affects its evaluation. These effects include temperature, the species and density of traps, clock frequencies, signal sizes, inter- and intra-spatial distributions, and subsequently capture and emission rates. A demonstration of the possible complications is shown by the following example. Five signal trains of six pixels each were defined. The first four are comprised of pixels which are either empty or contain a signal of 1620 e-'s. Input train A has a signal in pixel 1 only. B has signal in pixels 1 and 5. C has them in 1, 3, and 5. D is completely filled. The fifth train, E, has a Gaussian distribution whose sum total is equal to that of D. These trains were run through the model for the condition of 50 kHz, 521 transfers, -40 °C with a large-signal CTE of 0.99999. The results are shown in Table 1.

Table 1: CTE Model Results for Several Signal Trains

Train I.D.	Pixel 1	Pixel 2	Pixel 3	Pixel 4	Pixel 5	Pixel 6
A	1549	1.3	1.3	1.3	1.2	1.2
B	1549	1.3	1.3	1.3	1614	1.3
C	1549	1.3	1617	1.3	1618	1.3
D	1549	1619	1620	1620	1620	1620
Orig. E	210	1324	3326	3326	1324	210
E Out	200	1275	3258	3325	1325	212

First notice that the lead pixel result is the same in every train, A through D. It is likely that the rule of thumb of CTE to the N_{pix} power, which is usually applied to any CCD signal in order to estimate its degradation, actually only applies to the lead pixel of a signal train. Second, a "trap for the analyst" can be seen for Train A. Although the original signal has lost 71 e-'s, it would be difficult to detect it by measuring its tail, as is done with the EPER method. The emission time constant is calculated to be about 2.3 msec in this example, so the tail is stretched out so much that it would probably be lost in the read noise. Of course the main advantage of a Fe-55 CTE plot is that it is the charge <u>lost</u> by a single pixel event which is measured, and signal loss is easier to measure than the charge gained in image tails. A potential weakness in Fe-55 plots is suggested in Train B. Most of the traps encountered by the first pixel are still filled by the time the second signal pixel arrives. It would be easy to interpret that first pixel as a partial x-ray event and thereby to neglect it. As such, the computed CTE might be deceptively high, depending upon what the background for the application is. It is possible that Fe-55 CTE results in general are dependent upon the size and distribution of its background, i.e. regions comprised of anything that are not single pixel events. The idea is consistent with the knowledge that a fat zero background improves CTE for low-level signals. Finally, trains C and D simply show what effect signal density might have.

Train D lost 72 e-s after 521 transfers, but Train E lost 125 under the same conditions, although its sum equals that of D. The reason for this is that E has two pixels with about 3200 e-'s, which is about twice that of any D pixel, so they will see almost twice as many traps. This effect is compounded by the fact that its predecessors filled fewer traps than those of D. Internal signal distribution within a spot is almost certainly a significant factor.

4.0 Summary

One product of this study was a method by which pointing accuracy, precision and distortion could be estimated from either CTE measurements or projections from radiation damage models. This method is currently being applied to a funded project, and if successful, its results will probably be published later. Another product of this study is a CTE model which was the main subject of this article. The model tracks trap capture and emission rates based on signal size, particularly its volume. On an operational scale, the model tracks this charge exchange with traps over a selected number of pixel transfers. The model suggests several complicating factors which affect CTE evaluation, could inflate results, and make precise comparisons between experiments difficult.

5.0 References

[1] A.M. Mohsen and M.F. Tompsett, "The Effects of Bulk Traps on the Performance of Bulk Channel [CCDs]", IEEE Trans on ED, v. ED-21, Nov. 1974, pages 701 - 12.

[2] A.D. Holland, "The Effect of Bulk Traps in Proton Irradiated EEV CCDs", Nuc. Instr. & Meth., A326, 1993, pages 335 - 43.

[3] J.R. Janesick, "Short Course Notes: Advanced Scientific Charge-Coupled Devices", SPIE Printing, 1991, page 118 - 22.

[4] C. Kim, "The Physics of Charge-Coupled Devices" in Charge-Coupled Devices and Systems, edited by M.J. Howes and D.V Morgan, Wiley-Interscience, 1979.

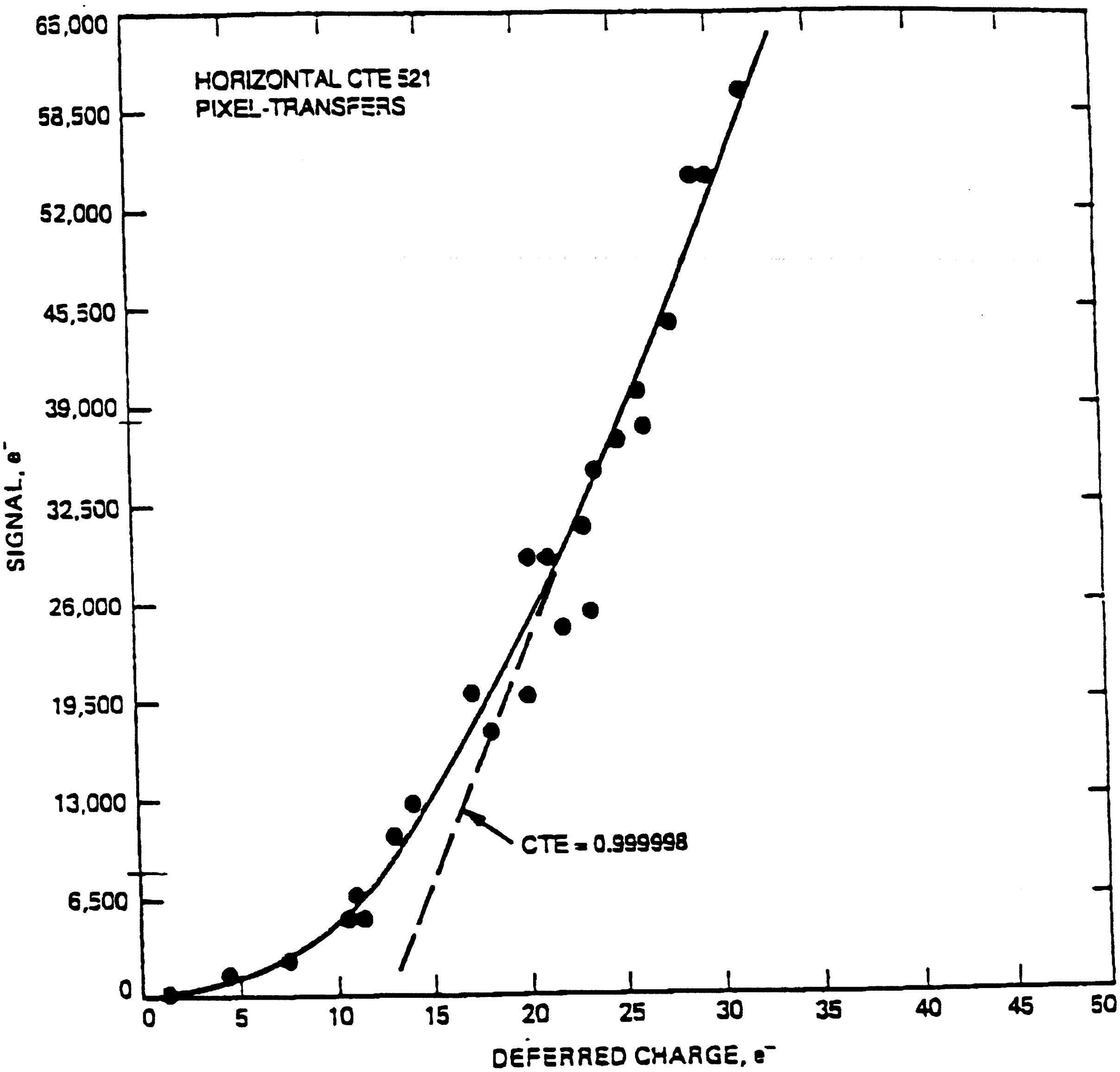

Figure 1: Effects of CTI in terms of the amount of deferred charge at various signal levels. Plot is from Ref. [3], J.R. Janesick's "Short Course Notes: Advanced Scientific Charge-Coupled Devices" (1991). Used with permission.

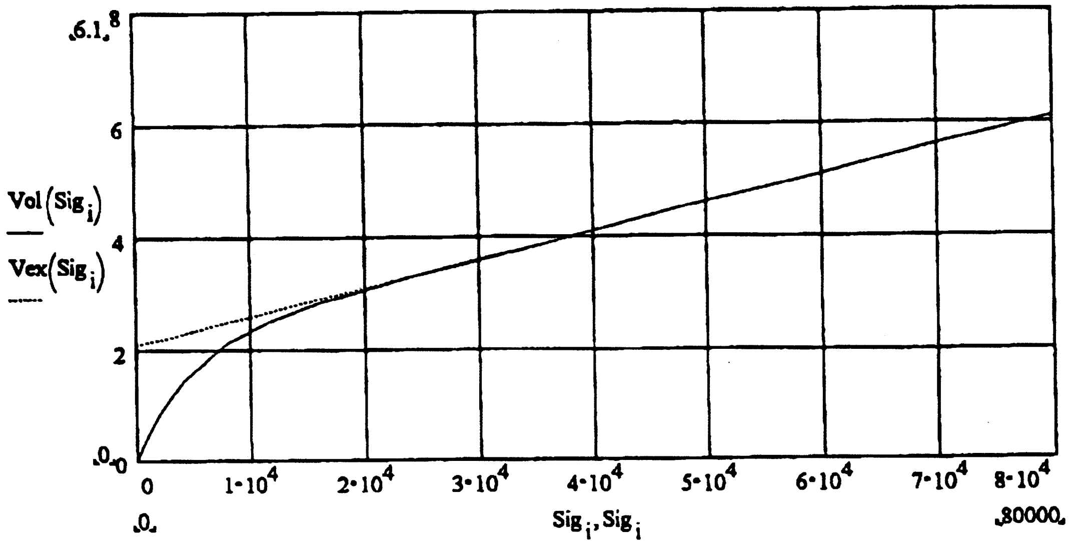

Figure 2: Signal volume (μm^3) versus signal charge count (e-'s) as described by Eqn (6). The ordinate intercept of the linear extrapolation is V_o. S_o is about 4700 e-'s here.

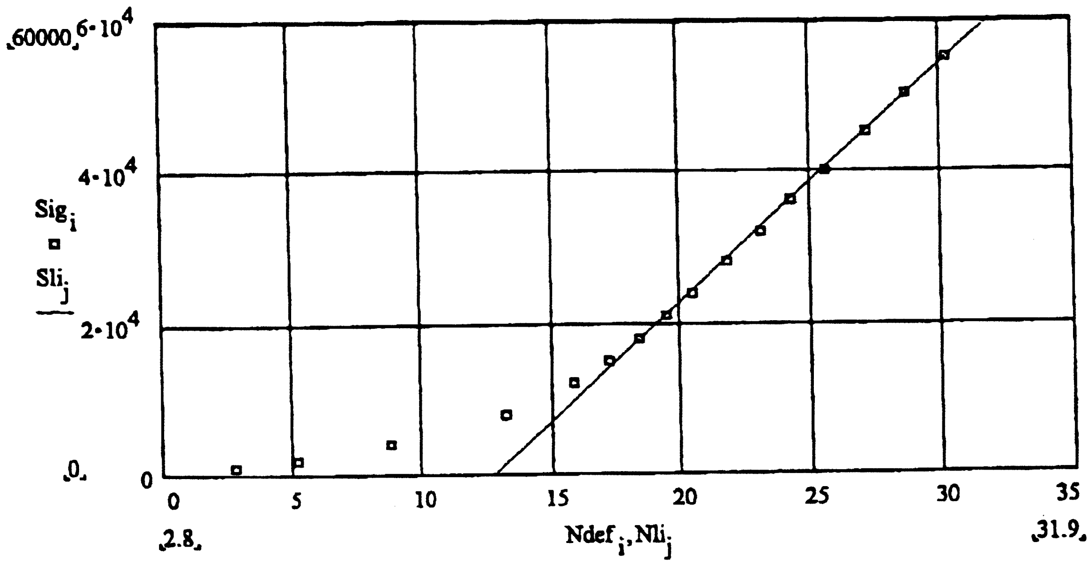

Figure 3: Results of CTE model, signal size (e-'s) versus the deferred charge count (e-'s) for the condition of CTI=6e-7, freq=50kHz, and 521 transfers. The axis orientation was chosen to facilitate comparison with Figure 1.

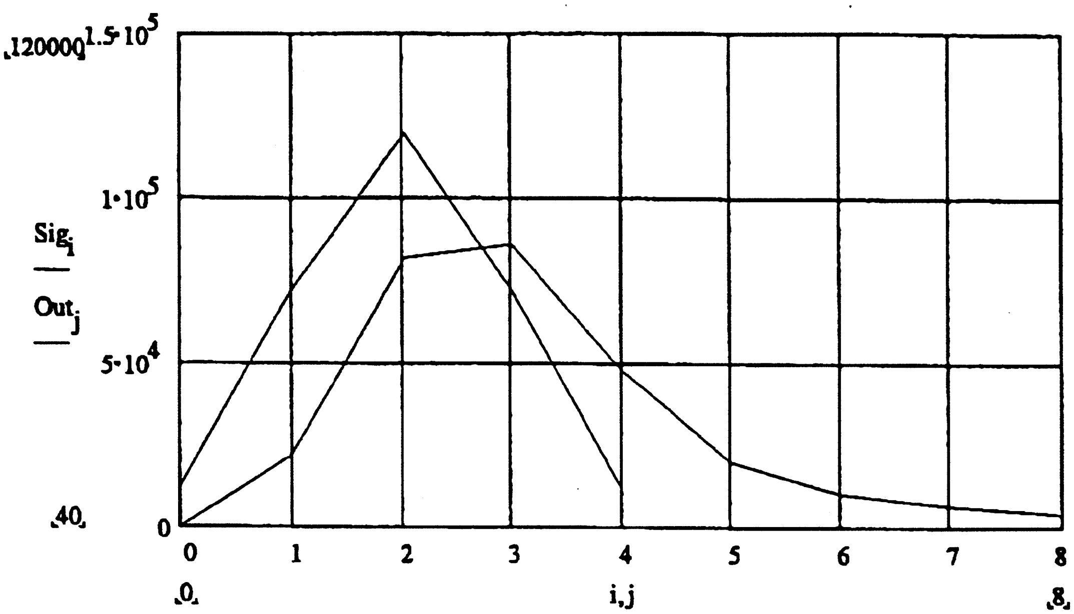

Figure 4: Model comparisons between an original imaged spot and its output after 1000 transfers at 50 kHz and -10 for a CCD with a CTE of 0.9993. Such a small CTE is very likely after years of operation under radiation exposure.